Bildregistrierung

- Goal and tasks of image registration
- Applications
- Image alignment transformations
- Extrinsic vs. intrinsic registration
- Intrinsic registration
  - Landmark-based image registration
  - Segmentation-based image registration
  - Pixel property based image registration
- Non-rigid image registration
- Image mosaicing
- Fusion without alignment
The big world

It’s a big world - so much to see

Joe Jackson, 'Big World' lyrics
The big world

It’s a big world - so much to see

Joe Jackson, 'Big World' lyrics

It’s a big world - TOO much to see ALL AT ONCE

See multiple times and combine the small views into a big view (image registration)
Image registration: Overlay for fusion

Nine near IR images of an urban area → fused image
Goal:

- **Alignment**: Calculation of the *spatial transform* that maps points from one image to corresponding points of another image.

- **Overlay**: Placement of two images in a common frame so that the information they contain can be optimally integrated or compared.
Applications

- Different views (multiview analysis; mosaic construction)
- Different sensor modalities (multimodal analysis)
- Different times (temporal analysis)
- Histological series (object reconstruction)

In many applications these factors appear simultaneously
Application: Multiview analysis (mosaic construction)

mosaicing human retina
Application: Multiview analysis (mosaic construction)

Happy Buddha – from Original to 3D hardcopy
http://www.cs.washington.edu/homes/curless/

(a) picture; (b) range image (Cyberware scanner); (c) fusion of 48 scans with 2.4 millions of polygons; (d) fusion of 58 scans (reduction to 800,000 polygons); (e) reproduced 3D model.
Application: Multimodal analysis (overlay for comparison)

- **Anatomical** knowledge from Magnetic Resonance Imaging (MRI)
- **Physiological/functional** knowledge from Single Photon Emission Computed Tomography (SPECT)
- Registered to have both types of knowledge simultaneously

Comparison of anatomical and functional images of the patient’s body can lead to a diagnosis, which would be impossible to gain otherwise.
Application: Multimodal analysis (overlay for comparison)

Multisensor registration of ETM+ at 30m (surround) to IKONOS at 1m (inset)
Application: Temporal analysis

Segmentation of three X-ray images (consecutive visits)
Application: Histological series

histological slices

Preregistered
(slices randomly oriented)

postregistered
Image registration

Tasks:

- **Alignment:**
  - ✓ Which spatial transformation to use?
  - ✓ How to find the optimal transformation?
  - ✓ How to measure the similarity between two images?

- **Overlay:**
  - ✓ How to blend image intensities (creating a seamless boundary between the images)
Alignment transformation

Nature of transformation:
- rigid (rotate, translate)
- similarity (rigid + scaling)
- affine (rigid + scaling & shear)
- projective
- non-rigid (curved, elastic)

Domain of transformation:
- global: apply to entire image
- local: subimages have their own transforms
Alignment transformation

- similarity
- affine
- perspective
- elastic
Determining the alignment transformation

- **Landmark-based methods**
  - Anatomical: Salient and accurately locatable points of anatomy
  - Geometrical: Points of some geometric property (local curvature extrema, corners, etc)

- **Segmentation-based methods**
  Identical anatomic structures, e.g. surfaces, are extracted from both images and used for the alignment procedure

- **Pixel property based methods**
  Operate directly on image grey values by minimizing some dissimilarity function over entire image
Landmark-based methods

**Fundamental:** A small number of corresponding points from two images suffice to determine their transformation

- Detection of feature points
- Correspondence analysis: Matching of feature points (using local image properties)
- Determination of the transformation from the matched feature points (robust estimation methods)
Landmark-based methods

Fundamental approach:

- Detection of feature points
- Matching of feature points
- Determination of the transformation from the matched feature points
- Geometric image transformation including interpolation
- Postprocessing (fusion of transformed images)
Segmentation-based methods

Example: Registration of medical images by matching tree structures (the vessel systems of liver and lung, e.g. bronchi and portal veins of liver)
Pixel property based methods

Parametric approach:

- no explicit landmark detection or segmentation
- explicit assumption of transformation model based on parameter set $p$; Example:
  - 2D rigid transformation: 3 parameters (1 for rotation and 2 for translation)
  - 3D rigid transformation: 6 parameters (3 for rotation around the three coordinate axes and 3 for translation)
  - 2D affine transformation: 6 parameters

- Search for the optimal solution in the transformation space
Pixel property based methods

2D rigid transformation

→ 3D optimization space

Where is the optimal transformation (point)?
Pixel property based methods

Registration problem: two images $f$ and $g$

$$\arg\min_p d(f_p, q) \text{ or } \arg\max_p s(f_p, q)$$

- $f_p$: $f$ transformed with transformation $p$
- $d()$: dissimilarity function of two images
- $s()$: similarity function of two images
Search for **optimal transform** in the space of transformations

- **Metric** - determines the “fitness” of the current registration iteration
- **Optimizer** - adjusts the transformation in an attempt to improve the metric
- **Interpolator** - applies transformation to image and computes subpixel values

Source: ITK registration guide
Pixel property based methods

(Dis)similarity metrics of two images $f$ and $g$:

- **Sum of squared differences (SSD):**
  
  $$d(f, g) = \frac{1}{N} \cdot \sum_{i} \sum_{j} [f(i, j) - g(i, j)]^2$$

  $N =$ size of overlap domain

  very sensitive to a small number of pixels that have very large intensity differences

- **Sum of absolute differences (SAD):**
  
  $$d(f, g) = \frac{1}{N} \cdot \sum_{i} \sum_{j} |f(i, j) - g(i, j)|$$
Pixel property based methods

(erbaut von Fabian Gigengack)
Pixel property based methods

Assumption SSD/SAD: after registration the two images differ only by Gaussian noise

- Normalized correlation coefficient (NCC)

\[
d(f, g) = \frac{\sum_{i} \sum_{j} [f(i, j) - \bar{f}] \cdot [g(i, j) - \bar{g}]}{\sqrt{\sum_{i} \sum_{j} (f(i, j) - \bar{f})^2} \sqrt{\sum_{i} \sum_{j} (g(i, j) - \bar{g})^2}}
\]

A less strict assumption of linear relationship between intensity values
Pixel property based methods

- Joint entropy

\[
H(f, g) = - \sum_{x=0}^{255} \sum_{y=0}^{255} P_{fg}(x, y) \log P_{fg}(x, y)
\]

\(P_{fg}(x, y)\): Joint probability distribution; probability of intensity \(x\) in image \(f\) and intensity \(y\) in image \(g\) at the same pixel position; it can be easily computed from histogram.

If the images are perfectly aligned, then the histogram is highly focused. As the images mis-align the dispersion grows. Entropy is a measure of histogram dispersion \(\rightarrow\) images are registered when one is transformed relative to the other to minimize the joint entropy.
Pixel property based methods

Example: Joint histograms of a MR image with itself. The leftmost histogram shows the situation when the images are registered. Because the images are identical, all grey value correspondences lie on the diagonal. The three following are the resulting histograms when one MR image is rotated with respect to the other by angles of 2, 5, and 10 degrees. Below the histograms are the corresponding joint entropy values.

![Joint histograms of a MR image](image)

<table>
<thead>
<tr>
<th>Entropy Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.82</td>
</tr>
<tr>
<td>6.79</td>
</tr>
<tr>
<td>6.98</td>
</tr>
<tr>
<td>7.15</td>
</tr>
</tbody>
</table>
Pixel property based methods

- Mutual Information (multimodal analysis)

\[
I(f, g) = H(f) + H(g) - H(f, g) = \sum_{x=0}^{255} \sum_{y=0}^{255} P_{fg}(x, y) \log \frac{P_{fg}(x, y)}{P_f(x)P_g(y)}
\]

- \( H(f) \): Entropy of \( f \)
- \( P_f(x) \): Probability of intensity \( x \) in image \( f \)
- \( P_g(y) \): Probability of intensity \( y \) in image \( g \)
- All terms can be easily computed from histogram

Mutual information from the information theory is a measure of statistical dependency between two data sets and particularly suitable for registration of images from different modalities. \( I(f, g) \) will be maximized when the images are aligned.

Pixel property based methods

Example: Mutual information

MI criterion (bottom row) computed in the neighborhood of point $P$ between new and old photograph (top row). Maximum of MI shows the correct matching position (point $A$). Point $B$ indicates the false matching position selected previously by the human operator. The mistake was caused by poor image quality and by complex nature of the image degradations.
Pixel property based methods

Example: Mutual information
Pixel property based methods

Example: Mutual information (cont.)

Joint entropy = 7.48  M.I. = 3.59
Pixel property based methods

Example: Mutual information (cont.)

reference image
current image
difference image
joint entropy = 9.36  M.I. = 1.70
Pixel property based methods

Joint entropy vs. mutual information:
Both measures are **computed for the overlapping parts** of the images and are therefore sensitive to the size and the contents of overlap.

**Example:** Search for optimal 1D vertical translation

- **Solution 1:**

![Image showing two overlapping images with a gray overlay for alignment]

Kapitel 5 “Bildregistrierung” – p. 34
Pixel property based methods

- Solution 2: better than solution 1 based on joint entropy

Mutual information is better equipped to avoid such problems because it includes the marginal entropies $H(f)$ and $H(g)$
Pixel property based methods

- Normalized gradient field (NGF; multimodal analysis)

\[ \text{NGF}(f, g) = - \sum_x \sum_y < n(f; x, y), n(g; x, y) >^2 \]

where \( n(f; x, y) \) is the normalized gradient at pixel \((x, y)\) of image \( f \):

\[ n(f; x, y) = \begin{cases} \frac{\nabla(f; x, y)}{|| \nabla(f; x, y) ||}, & \text{if } \nabla(f; x, y) \neq 0 \\ 0, & \text{otherwise} \end{cases} \]

It is assumed that the gradients of \( f \) and \( g \) match when they are perfectly aligned (see Kapitel “Kantendetektion“)
### (Dis)similarity metrics

<table>
<thead>
<tr>
<th>Measure</th>
<th>monomodal</th>
<th>multimodal</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSD</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>CC</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>NCC</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>MI</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>NGF</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Pixel property based methods

Optimization methods:
Finding the minimum of dissimilarity measure is a multidimensional optimization problem (#dimensions = degree of freedom of geometrical transformation). The only method yielding global extreme solution is an exhaustive search (computationally demanding!).

Sophisticated optimization algorithms are required, which help to localize the minima/maxima:
- Gauss-Newton numerical minimization algorithm
- Levenberg-Marquardt optimization method
- Powell’s multidimensional direction set method
- …..
Pixel property based methods

- The desired optimum when registering images using similarity measures is frequently not the global optimum, but one of the local optima (e.g. extremely small overlap cannot be our goal of registration).

- Start the optimization algorithm within the „capture range“ of the correct optimum, i.e. within the portion of the parameter space, in which the algorithm is more likely to converge to the correct optimum than the incorrect global one.
### Image registration software tools

<table>
<thead>
<tr>
<th>Software</th>
<th>Source</th>
<th>Language</th>
<th>Platform</th>
<th>GUI</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITK</td>
<td>√</td>
<td>C++</td>
<td>Linux, PC, MAC</td>
<td>–</td>
<td>√</td>
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<tr>
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<td>Linux, PC, MAC</td>
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<td>–</td>
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<tr>
<td>ART</td>
<td>–</td>
<td>C++</td>
<td>Linux, PC, MAC</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

- FSL (FMRIB Software Library): [http://www.fmrib.ox.ac.uk/fsl/](http://www.fmrib.ox.ac.uk/fsl/)
Intrinsic vs. extrinsic registration

Intrinsic methods (all methods discussed so far):
Based on image information as generated by the imaged object (e.g. patient)

Extrinsic methods:
Based on *foreign objects* introduced into the imaged space (motion capture, medical imaging)
Extrinsic methods:
Based on foreign objects introduced into the imaged space

- **Advantages**
  - Registration is easy and fast, and can be automated
  - No need for complex optimization algorithms

- **Disadvantages**
  - Manual work needed in the pre-acquisition phase
  - Often invasive character of the marker objects
Non-rigid registration
Non-rigid registration

- **Modeling of motion as physical process**
  Describe accurately the deformation field as a physical phenomena (e.g. stretching of an elastic material such as rubber). The partial differential equations (PDE) describing the corresponding processes are solved by numerical integration techniques.

- **Optical flow techniques**
  Originally used in computer vision to recover the relative motion between two successive frames of an image sequence (Kapitel “Bewegungsanalyse: Optical Flow”)

- **Approaches based on control points**
  Thin-plate splines, B-splines, etc.
Non-rigid registration

**Optical flow**: Find a *dense field of deformation vectors* (huge transformation space). This field maps each point from one image into the corresponding position in the other image.

How to describe the motion between the two frames?
Non-rigid registration

Optical Flow

$I(t), \{p_i\}$

$I(t + 1)$

Velocity vectors \(\{v_i\}\)
Image mosaicing

Process of seamlessly stitching together or blending a set of overlapping images of a scene into one large image

Approaches:

- **Sequential (local)**
  One simple way to register a large number of images is to add new images to the panorama one at a time, aligning the most recent image with the previous ones already in the collection. In the case of 360° panoramas, accumulated error may lead to the presence of a gap (or excessive overlap) between the two ends of the panorama.

- **Parallel (global)**
  A better alternative is to simultaneously align all the images together to correctly distribute any mis-registration errors. This process of simultaneously adjusting transformations for a large collection of overlapping images is called bundle adjustment in the photogrammetry community.
Image mosaicing

Local approach: mosaicing = registering two images

- Determination of the transformation between the two images
  - Select a suitable type of transformation
  - Determine a sufficient number of corresponding control points
  - Use the control points to compute the transformation, which spatially aligns the two images

- Blending image intensities
  Blend intensities in overlapping areas of the two images such that they seamlessly merge into one larger image. Intensities of corresponding pixels in overlapping areas may be different.
Image mosaicing

Selection of transformation:

- Translation
  - Camera with a fixed lens center and a horizontal optical axis while rotating the camera about a vertical axis passing through the lens center
  - Camera is sufficiently far from the scene

- Translation on cylindrical surface
  Camera as above, but not very far from the scene. Images obtained during the rotation can be mapped to a cylindrical surface whose axis is parallel to the rotation axis.

- Projective transformation
  - Camera not fixed and not far from the scene
  - Scene is flat
Image mosaicing

Blending image intensities:

Needed so that if overlapping areas in the two images have different intensities, intensities in one image gradually convert to intensities in the other image, creating a seamless boundary between the images.

\[
I_p = \frac{d_B \cdot I_p^A + d_A \cdot I_p^B}{d_A + d_B}
\]

The weights \(d_A\) and \(d_B\) are set inversely proportional to the distances of \(p \in C\) to the closest border pixels.
Image mosaicing

Example: The images exhibit considerable intensity differences, due to the fast moving clouds and change in scene illumination. The intensity blending process has gradually converted intensities in one image to intensities in adjacent images.
Averaged faces (atlas):
- Take a large number of faces of the same gender
- Register (align and overlay) them to build an average face
Why are the resulting average faces generally beautiful?

- One reason might be that by calculating average proportions, unpleasant asymmetries and irregularities become levelled out.

- By blending together several faces wrinkles and pimples gradually disappear → The skin looks younger and perfectly smooth.
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Joe Jackson, 'Big World' lyrics

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See multiple times and combine the small views into a big view (image registration)

Look at local (spatially, temporally, etc.) and see global
References

1993–2003 >> 1000 papers on registration:


J. Modersitzki: Numerical Methods for Image Registration, Oxford University Press, 2004


Fusion without alignment

Fusion of multi-exposure images
Fusion without alignment

Fusion of multi-focus images